Finite-temperature chiral transitions in QCD with the Wilson quark action *

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We investigate the finite-temperature phase structure and the scaling of the chiral condensate in lattice QCD with two degenerate light quarks, using a renormalization group improved gauge action and the Wilson quark action. We obtain a phase diagram which is consistent with that from the parity-flavor breaking scenario. The scaling relation for the chiral condensate assuming the critical exponents and the scaling function of the three dimensional O(4) model is remarkably satisfied for a wide range of parameters. This indicates that the chiral transition in two flavor QCD is of second order in the continuum limit.

1. Introduction

As a step toward the clarification of the finite-temperature QCD transition, it is important to investigate the nature of the transition on the lattice with two degenerate light quarks. In a previous study[1], using the Wilson quark action and a renormalization group (RG) improved gauge action

$$S_g^R = \frac{\beta}{6}\left(c_0 \sum W_{1\times1} + c_1 \sum W_{1\times2}\right), \qquad (1)$$

with $c_0 = 1-8c_1$ and $c_1 = -0.331$ [2], we studied the nature of the phase transition and the scaling behavior of the chiral condensate at $\beta > \beta_{\rm ct}$ ($\approx$ 1.35) on an $N_t = 4$ lattice, where $\beta_{\rm ct}$ is the value of β at the chiral transition point. In the present work, we extend the study to $\beta \le \beta_{\rm ct}$ performing simulations at $\beta = 1.1$, 1.2, and 1.35 on an $8^3 \times 4$ lattice. We also determine the phase structure of the chiral limit, in particular, for $\beta \le \beta_{\rm ct}$ at $N_t = 4$ on a (β, K) plane.

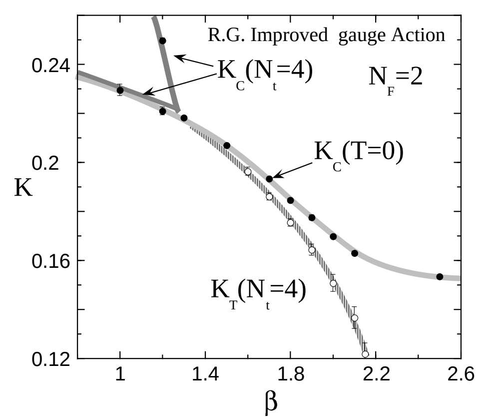

Figure 1. Phase diagram for $N_F = 2$ with the RG improved gauge action and the Wilson quark action.

2. Phase diagram

Fig.1 shows our result for the phase diagram on a (β, K) plane. In the previous work[1], the line of the zero-temperature chiral limit $K_c(T = 0)$ defined by the vanishing point of the pion mass was determined on an 8^4 lattice, and the finite-temperature transition/crossover line K_t for $N_t =$

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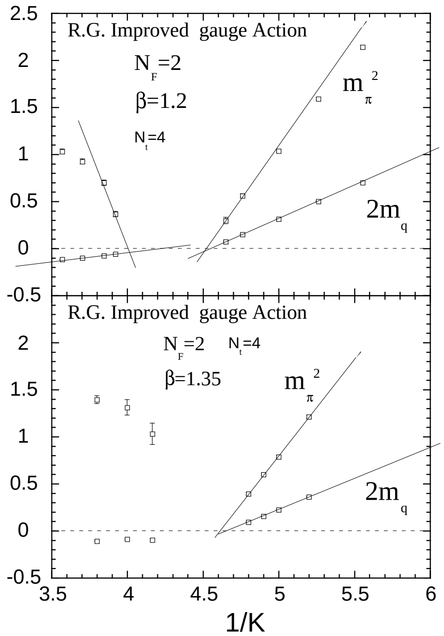

Figure 2. m_π^2 and $2m_q$ vs. 1/K for $N_F = 2$ obtained at $\beta = 1.2$ and 1.35 on an $8^3 \times 4$ lattice.

4 was determined on an $8^3 \times 4$ lattice. The K_t line crosses the $K_c(T = 0)$ line at $\beta_{\rm ct} \approx 1.35$.

In this work, we estimate the location of the finite-temperature chiral limit $K_c(N_t = 4)$ for $N_t = 4$, defined by the vanishing point of the pion screening mass m_π. In Fig.2, we plot m_π^2 and the quark mass m_q for $N_t = 4$ at fixed β as a function of $1/K$, where m_q is defined by an axial Ward identity[3,4]. In the upper figure for $\beta = 1.2$, there are two chiral limits. On the other hand, at $\beta = 1.35 \approx \beta_{\rm ct}$ in the lower figure, we cannot find a clear gap between the two chiral limits. These results imply that the line $K_c(N_t = 4)$ for the chiral limit turns back towards strong coupling around $\beta_{\rm ct}$, forming a cusp, as shown in Fig.1. This structure is consistent with that expected from the parity-flavor breaking scenario[5], which was confirmed for the standard one plaquette gauge action and the Wilson quark action[6].

3. Scaling analysis

The magnetization M near the second order transition is expected to be described by a single

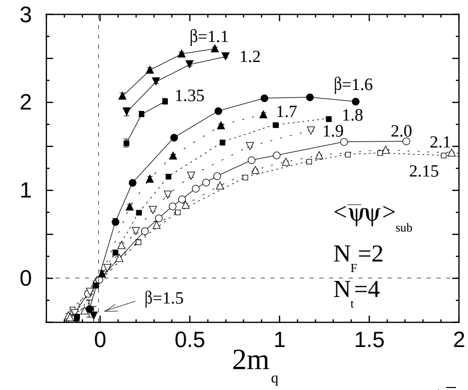

Figure 3. Subtracted chiral condensate $\langle\bar\Psi\Psi\rangle_{\rm sub}$ as a function of $2m_q a$.

scaling function;

$$M/h^{1/\delta} = f(t/h^{1/(\beta\delta)}), \tag{2}$$

with h the external magnetic field and $t = [T - T_c(h{=}0)]/T_c(h{=}0)$ the reduced temperature.

According to the universality argument, when the chiral transition is of second order, two flavor QCD belongs to the same universality class as the three dimensional O(4) spin model[7], and the chiral condensate should satisfy the scaling relation (2) with the O(4) exponents and the O(4) scaling function. We identify $h = 2m_q a$, $t = \beta - \beta_{\rm ct}$, and $M = \langle\bar\Psi\Psi\rangle_{\rm sub}$, where M is a subtracted chiral condensate defined through an axial Ward identity[3],

$$\langle\bar\Psi\Psi\rangle_{\rm sub} = 2m_q a(2K)^2 \sum_x \langle\pi(x)\pi(0)\rangle. \tag{3}$$

Our results for $\langle\bar\Psi\Psi\rangle_{\rm sub}$ are shown in Fig.3.

We make a fit to the scaling function of the O(4) model by adjusting $\beta_{\rm ct}$ and the scales for t and h, with the exponents fixed to the O(4) values[8], including all the data in the range $0 < 2m_q a < 0.8$ and $\beta \leq 2.0$ shown in Fig.3. We note that the data for the lightest quark mass (m_q = 0.06-0.07) at β = 1.1, 1.2, and 1.35 are slightly off the scaling curve. Therefore, we make a fit excluding these three points. The result of the fit with $\chi^2/df = 0.72$ is shown in Fig.4(a). The result shows that scaling works well in the $t \leq 0$ region as well as the $t > 0$ region, except the three lightest quark mass data at $t < 0$, indicated

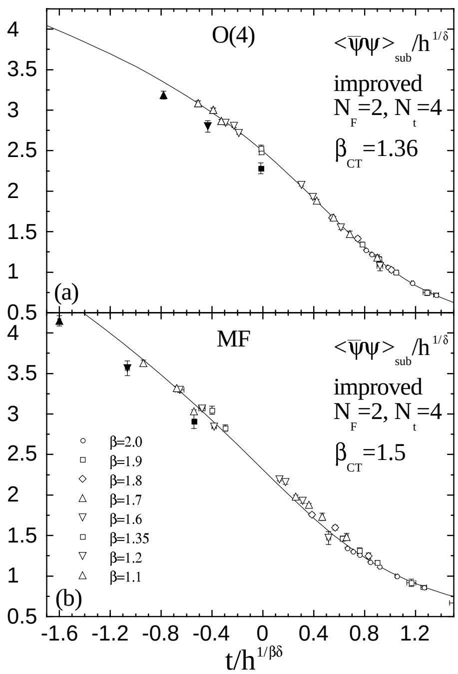

Figure 4. Fits to the scaling function with (a) O(4) and (b) MF exponents. Solid curves are scaling functions obtained in an O(4) spin model[9] and by a MF calculation, respectively.

by the filled symbols. The adjusted $\beta_{\rm ct}=1.36(1)$ is consistent with $\beta_{\rm ct}=1.35(1)$ obtained by the previous fit using only the $t>0$ data[1].

One possible origin of the deviation from the scaling curve of the lightest quark mass data at $t \leq 0$ is a finite size effect, because, in the confining phase, we expect that finite size effects are severe at small quark mass. To explore this possibility, we are performing simulations on $L^3 \times 4$ ($L=12$ and 16) lattices for the lightest quark mass data at $\beta=1.1, 1.2$ and 1.35. The statistics we have accumulated so far (about 50 configurations each) is not yet high enough to obtain a definite conclusion about the systematic size dependence of the deviation from the scaling curve.

Another possible origin for the deviation is the explicit chiral breaking due to the Wilson term, which is expected to be large at small β. It is plausible that this effect becomes visible as deviation from scaling, when the explicit chiral violation due to the Wilson term becomes larger than that from the quark mass. The fact that we observe the deviation from the scaling at the lightest quarks mass for small β region is consistent with this interpretation. To confirm this interpretation it is necessary to make a simulation at larger N_t to reach a larger β region, or use an improved quark action.

We also study the possibility of the MF scaling[10]. We perform the scaling test using MF exponents and the MF scaling function. Because m_π at β=1.5 does not vanish in the chiral limit (cf. Fig.3 in Ref.[1]), we restrict $\beta_{\rm ct} \leq 1.5$. We then obtain the best fit shown in Fig.4(b) with $\chi^2/df=6.2$, to be compared with $\chi^2/df=0.72$ obtained for the O(4) case [Fig.4(a)]. We exclude the MF scaling, as the data is much more scattered than in the O(4) case.

The success of the scaling with the O(4) exponents, albeit with the three lightest quark data at $t<0$ excluded, indicates that the chiral transition for two flavor QCD is of second order in the continuum limit.

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